Technical optimum milk and meat production levels in dual-purpose cattle systems in tropical Mexico

Yuridia Bautista Martínez a
José Antonio Espinosa García b*
José Guadalupe Herrera Haro c
Francisco Ernesto Martínez Castañeda d
Humberto Vaquera Huerta e
Benigno Estrada Drouaillet a
Lorenzo Danilo Granados Rivera e

a Universidad Autónoma de Tamaulipas. Facultad de Medicina Veterinaria y Zootecnia. Carretera Mante Km 5. 87000. Ciudad Victoria Tamaulipas, México.


e Campo Experimental General Terán, INIFAP. General Terán. Nuevo León, México.

*Corresponding author: espinosa.jose@inifap.gob.mx
Abstract:

Inputs directly affect profitability in livestock production, although what effects they have vary in response to production system and input type. An analysis was done of the results from a milk and meat production function using data from dual-purpose system (DP) production units in three locations in tropical Mexico. Data were collected through monthly surveys and covered milk production, meat production, income and financial costs over a twelve-month period. The functions were estimated by the indirect linear regression method with transformed data for a Cobb-Douglas function. The milk function showed the feed and cows inputs to explain 91% of production. Elasticity coefficients were 0.34 for feed and 0.5 for cows. Marginal products were 0.75 for milk and 892.2 for cows, with values of $4.03 L for milk and $4,800.20 per cow. Both inputs are in stage II of production with diminishing marginal returns. For meat production both the feed and cows’ inputs explained 72% of production, with elasticities of production coefficients of -0.20 for feed and 1.11 for cows. Feed was in stage III of production with negative marginal returns, but the cows input was in stage I with increasing marginal returns. The sum of the coefficients was less than one for both functions (0.92 for feed, 0.91 for cows), indicating decreasing returns to scale. The optimum technical production levels were 488.97 L milk per day and 10 calves per year. In the studied producers the inputs for milk production were being used rationally, although in meat production feed appears to be overused and should be evaluated.

Key words: Cobb-Douglas, Elasticity, Marginal product, Returns to scale; technical optimum.

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Introduction

A total of 11.8 billion liters of milk were produced domestically and 3.7 million liters imported to meet domestic demand in Mexico in 2017. For the same period domestic beef production was 1.85 million tons with 136,000 t imported to meet demand[1]. Dairy and beef producers in Mexico clearly do not generate enough product to meet domestic demand, highlighting the need for quantitative analysis of the efficiency of specialized, semi-specialized, dual-purpose and family dairy and beef production systems to optimize resource
use. Implemented mainly in tropical regions\(^2\), dual-purpose cattle systems (DP) are characterized by milk production coupled with sale of weaned calves for beef\(^3\). One of its main advantages is that feed costs are reduced since most systems are based on grazing with supplementation for lactating cows\(^2\).

A vital aspect of DPs is the need for efficiency analysis of production units to maximize appropriate use of inputs for production\(^4\). Parametric methodologies have been developed based on estimation of production functions to study the functioning of these systems and manifest cause-and-effect relationships in them. These methodologies identify the relationship between the amounts of different inputs and the quantities of resulting products, as well as associating each input with the maximum production level per period. This data can then be used to formulate productive development strategies for a specific region.

Physiological and non-physiological factors such as feed quantity and quality, fodders, herd size, season, and lactation number and stage, among others\(^5\), influence milk and beef production. It is therefore important to understand the factors that best explain production to facilitate selection of the inputs to be used and make optimal use of them\(^6\). The Cobb-Douglas function is widely used to identify production functions in livestock systems, and has been applied to estimate milk and beef production in different systems and regions in Mexico\(^7,8,9\). Indicators can be calculated using the properties of Cobb-Douglas type functions and the theory of production. Principal among these is elasticity of production, which is the percentage change in the amount produced relative to the percentage change in input levels\(^10\). Another indicator is marginal return, which describes production decreases or increases in response to addition of an input, and, depending on its behavior (i.e. increase, decrease, zero or negative), can indicate whether the input analyzed is in stage I, II or III of a classic production function. This indicator also allows identification of return types at the livestock production unit level, which helps to explain how production behaves in response to proportional and simultaneous variation of all inputs, which can be increasing, constant or decreasing. Input sales price is used to estimate the indicator marginal product, which is the variation in the quantity produced in response to unit increases in any production input \textit{(ceteris paribus)} as well as the marginal product value, which is the additional income earned by a livestock company for each additional input unit\(^11\). These data are useful for economic advisers in the livestock sector, extension representatives advising producers and producers themselves. They help in making decisions on rational resource use, and appropriate increases or decreases in inputs for the production process, all aimed at augmenting profits. The present study objective was to use production functions to analyze data from representative dual-purpose (DP) system milk and beef producers in the Mexican tropics to estimate those inputs that have the greatest influence on production, and calculate the technical optimum levels subject to input price and milk and meat sale price to determine if they are being used rationally.
Material and methods

Study area

The study was done in production units (PU) in three states representative of the tropics in Mexico, and where the DP system predominates. Production units in the state of Tabasco (17°51’ N; 93°23’ W) were at 2 m asl, in an area with a warm humid climate and abundant summer rains, a 26.4 °C average annual temperature and 190.85 mm mean monthly rainfall. The units in Chiapas (15°41’12” N; 93°12’33” W) were at 57 m asl in an area with a warm subhumid climate, 28 °C average annual temperature, and 80 mm mean monthly rainfall. In Sinaloa (23°14’29” N; 106°24’35” W) the units were at 10 m asl, in an area with 26.0 °C average annual temperature, and 63 mm mean monthly rainfall(12). All PU used Bos indicus x Bos taurus animals. Feeding was based on extensive grazing using supplementation with commercial balanced diets based on net lactation energy and 17 % protein during lactation for tall and medium-height cows. The average number of producing cows among the PU was 39. These were milked once a day using the calf to stimulate milk flow, extracting three-quarters of the udder for sale and leaving a quarter to feed the calf. Meat production in all units consisted of the sale of calves weaned at 160 kg average weight.

Input classification and productive variables

Data were collected via monthly producer surveys from June 2012 to July 2013, in 30 PU, 10 per state. Production unit (PU) selection was done by unrestricted random sampling from the PU registered in cooperating local cattle associations. The surveys consisted of forms with sections on herd structure, land use, income from sale of milk and meat, and expenses from supply purchases. The variables used in the production function were based on recommendations for estimating problems when a producer generates multiple products such as livestock and agricultural crops, and when these can change substantially from one region to another; for example, the quantities of concentrate feed, animals, labor and fuel(13). A total of ten variables were recorded: total annual milk production (liters); total annual calf production; total amount of concentrate feed used in the PU per year (kg); number of producing cows; grazing area (hectares); preserved fodder used in PU per year (kg); full-time and seasonal labor (days); number of sires; operating supply costs (electricity, gasoline, diesel).
**Data analysis**

Milk and meat production functions were estimated using the indirect method to generate a Cobb-Douglas type function, which consists of a linear regression with the original data transformed to Neperian logarithms of the dependent and independent variables\(^6\).

After each of the variables was converted, the model that best explains milk and meat production was selected using the STEPWISE procedure in the SAS program. This procedure begins by calculating the simple correlation matrix, based on the correlation values; the independent variable (Xi) with the highest correlation to the response variable (Yi) is included in the model. Selection of the variable to include in the model was done using the partial correlation coefficients (\(R^2\)). At each step the contribution of each variable to the model is examined by applying the partial F test as a criterion; therefore, at each stage all variables are examined for their unique contribution to the model, and those that do not meet a previously established criterion are eliminated.

The estimated specific model for milk is:

\[
\ln Y_1 = \beta_0 + \beta_1 \ln X_1 + \beta_2 \ln X_2 + \beta_3 \ln X_3 + \beta_4 \ln X_4 + \beta_5 \ln X_5 + \beta_6 \ln X_6 + \beta_7 \ln X_7 + \varepsilon
\]

The estimated specific model for meat is:

\[
\ln Y_2 = \beta_0 + \beta_1 \ln X_1 + \beta_2 \ln X_2 + \beta_3 \ln X_3 + \beta_4 \ln X_4 + \beta_5 \ln X_5 + \beta_6 \ln X_6 + \beta_7 \ln X_7 + \varepsilon
\]

Where:

\(Y_1\) = milk production;

\(Y_2\) = annual calf production;

\(X_1\) = concentrated feed used in PU in kg yr\(^{-1}\);

\(X_2\) = lactating cows;

\(X_3\) = grazed area in hectares;

\(X_4\) = preserved forage used in PU in kg yr\(^{-1}\);
\( X_5 = \text{labor;} \)
\( X_6 = \text{sires;} \)
\( X_7 = \text{operating supply costs (electricity, gasoline, diesel);} \)
\( \beta_i = \text{parameters to be estimated (i = 0, 1, \ldots,7);} \)
\( \varepsilon = \text{residual term.} \)

After estimating the Cobb-Douglas function with the variables that best explain meat and milk production, the input coefficient values were used to calculate the elasticity of production, marginal returns and the production stage of each input. In a Cobb-Douglas function, each input’s coefficient value is equal to its elasticity of production. If this is greater than 1 the input has increasing marginal returns and if it is less than 1 it has diminishing marginal returns. In addition, each input’s elasticity of production indicates the production stage in which it is located: a value \( \beta >1 \) indicates stage I; \( \beta <1 \) is stage II; and \( \beta <0 \) is stage III\(^{(11)}\). The type of returns to scale of the studied livestock producers was identified using the sum of the input coefficients of the milk and meat production functions. Calculations were also done of the marginal product (PMgXi) and the value of the marginal product (VPMgXi) of the inputs derived from the elasticity formula using the means of total milk production and of the inputs, with the following formulas\(^{(14)}\).

\[
\text{Ep}(b_1) = \frac{\partial Y/Y}{\partial X_i/X_i} = \frac{X_i \partial Y}{Y \partial X_i} = \frac{\text{PMgX}_i}{\text{PPX}_i}
\]

\[
\text{PMgX}_i = \text{Ep}(b_1) \times \text{PPX}_i
\]

\[
\text{VPMgX}_i = \text{PMgX}_i \times P_{Y_i}
\]

Where,
\( \text{PMgX}_i = \text{Marginal product of input } X_i \)
\( \text{Ep}(b_1) = \text{Elasticity of } Y_i \)
\( Y_i = \text{Mean of annual milk or calf production} \)
\( X_i = \text{Mean of input used} \)
\( \text{VPMgX}_i = \text{Value of marginal product } X_i \)
\( P_{Y_i} = \text{Unit price of } Y_i \)
\( \text{PPX}_i = \text{Average product of input } X_i \text{ used} \)
The average product of each input was the ratio between mean production (milk, calves) and input average (feed, cows).

Technical optimum milk and meat production levels were estimated by the Lagrange multiplier method, optimizing the milk and meat production functions (objective functions), subject to the prices of the inputs used and product sale price (one liter of milk and one calf).

\[ L = f(X_1, X_2) - \lambda (P_X_1 + P_X_2 + M) \]

Where,
- \( L \) = Lagrange Function
- \( \lambda \) = Lagrange Multiplier
- \( F(X_1, X_2) \) = Cobb-Douglas production function for milk and meat
- \( P_X_1, P_X_2 \) = Price of variable inputs
- \( M \) = Product unit price

The algebraic procedure consisted of subtracting the constraint from the objective function, and \( L \) (first order condition) was partially derived from \( X_1, X_2 \), and \( \lambda \). Using the maximization rule, the ratio of partial derivatives was equalized to \( X_1 \) and \( X_2 \), which were limited to the input price ratio. The solution provided the values of \( X_1 \) and \( X_2 \), which were substituted in the Cobb-Douglas function, thus estimating the technical optimum levels for milk (in liters) and calf production.

Average sale price for a liter of milk was $ 5.38 and that for calves was $ 6,020. The average cost of one kilo feed was $ 4.00. The cost of a producing cow cost was estimated using the capital recovery formula\(^{(15)}\), where purchase cost of a replacement heifer was $ 18,000.00, assumed use life was 8 yr in a DP system, annual return rate was 12.5 %, and estimated cost of one cow per year was $ 500.00. Calculation of the technical optimum level in the milk production function was done considering the price of one cow per day; that is the quotient of the price of one cow per year / 365 d, which was $ 1.36.
Results and discussion

Herd structure in the studied PU varied with production intensity and the area available for livestock within the PU (Table 1). Constant movement also occurred due to cow physiological condition (heifer, dry, lactating) or animal purchase and sale\(^{16}\).

<table>
<thead>
<tr>
<th>Variable</th>
<th>n</th>
<th>Mean ((\bar{y}))</th>
<th>SD (S)</th>
<th>CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Producing cows</td>
<td>30</td>
<td>38.9</td>
<td>15.19</td>
<td>38.81</td>
</tr>
<tr>
<td>Dry cows</td>
<td>30</td>
<td>18.97</td>
<td>9.91</td>
<td>52.24</td>
</tr>
<tr>
<td>Heifers</td>
<td>30</td>
<td>19.80</td>
<td>11.71</td>
<td>59.14</td>
</tr>
<tr>
<td>Bull calf</td>
<td>30</td>
<td>14.22</td>
<td>8.54</td>
<td>60.05</td>
</tr>
<tr>
<td>Cow calf</td>
<td>30</td>
<td>12.40</td>
<td>6.73</td>
<td>54.27</td>
</tr>
<tr>
<td>Sires</td>
<td>30</td>
<td>2.49</td>
<td>1.49</td>
<td>59.83</td>
</tr>
</tbody>
</table>

\(n=\) number of production units; \(SD=\) standard deviation; \(CV=\) coefficient of variation.

In the milk and meat production function model, the coefficient of determination (\(R^2\)) indicates that most of the variability in production is explained by the independent variables Feed (91.9\%) and cows (72.4\%) (Table 2). The percentage of unexplained variation in both models can be attributed to differences between PU such as herd management practices or environmental conditions. In milk production systems the feed input explains a greater percentage of variation in milk production than other inputs\(^{15,17}\). As a result, fresh fodder, preserved fodder and concentrate feed can be used strategically to increase milk production\(^{5,18}\). Feed handling and quality is clearly important in dairy production systems since it is directly related to production.

Average herd size was 39 producing cows, average annual milk production was 93,678.5 L, and average annual calf production was 14 (Table 3).
Table 2: Regression models selected for milk and meat production

<table>
<thead>
<tr>
<th></th>
<th>Estimated parameter</th>
<th>Standard error</th>
<th>Pr &gt;F</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Milk</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>6.099</td>
<td>0.310</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>InX₁</td>
<td>0.346</td>
<td>0.034</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>In X₂</td>
<td>0.542</td>
<td>0.095</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>R²</td>
<td>0.919</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Σb₁ = 0.888</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

|          |                     |                |       |
| **Meat** |                     |                |       |
| Intercept| 0.853               | 0.430          | 0.0579|
| InX₁     | -0.205              | 0.047          | 0.0002|
| In X₂    | 1.118               | 0.133          | <.0001|
| R²       | 0.724               |                |       |
| Σb₁ = 0.913 |                  |                |       |

InX₁ = Neperian logarithm of kilograms feed concentrate; In X₂ = Neperian logarithm of producing cows; R² = coefficient of determination; Σb₁ = sum of b₁ coefficients.

Table 3: Means for annual milk and calf production, inputs used in double-purpose system production units

<table>
<thead>
<tr>
<th>Function</th>
<th>MPL</th>
<th>MPB</th>
<th>MIA</th>
<th>MIV</th>
<th>PPA</th>
<th>PPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milk</td>
<td>93,678.5</td>
<td>--------</td>
<td>45,678.9</td>
<td>38.9</td>
<td>2.05</td>
<td>2,408.18</td>
</tr>
<tr>
<td>Meat</td>
<td>--------</td>
<td>14.22</td>
<td>45,678.9</td>
<td>38.9</td>
<td>3.11X10⁻⁰⁴</td>
<td>0.365</td>
</tr>
</tbody>
</table>

MPL= mean milk production kg PU yr⁻¹; MPB= mean calf production heads PU yr⁻¹; MIA= mean feed input kg PU yr⁻¹; MIV= mean cow input head PU yr⁻¹; PPA= average product of feed input; PPV= average product of cow input.

Cobb-Douglas production function for milk

\[\ln Y_1 = 6.099 + 0.346 \ln X_1 + 0.542 \ln X_2 \text{ (Equation 1)}\]

After transformation with antilogarithms:

\[\text{Milk} = e^{6.099} X_1^{0.346} X_2^{0.542} \text{ (Equation 2)}\]

\[\text{Milk} = 445.69 X_1^{0.346} X_2^{0.542} \text{ (Equation 3)}\]
Elasticities of production for milk

In the present results a 1% increase in the X₁ input (feed) raised total milk production by 0.34 % (ceteris paribus). A 1% increase in the X₂ input (cows) raised total milk production by 0.54 %. Increases in either input positively affected milk production, although increasing the number of cows provided a better response in production than increasing feed. This coincides with reports from other dairy production systems in tropical climates similar to those of the study area in which the cow input provided the most elasticity in the studied inputs, with values ranging from 0.40 to 0.60 %, whereas the feed input produced elasticities of 0.15 to 0.30 % (14). Understanding to what degree inputs impact production is vital when analyzing livestock producers since under certain circumstances increases in inputs can decrease production, resulting in negative elasticities (10). For example, dairies in the tropics of India report a 2.4 % decrease in milk production as feed intake increases (19), resulting in financial losses to producers.

Marginal returns and input production stage for milk production

The elasticity coefficients for the milk production function were 0.34 for feed and 0.54 for cows (Equation 1). According to the law of marginal returns both inputs have diminishing marginal returns because their values are less than 1. Also, they are in stage II of a classic production function, meaning that increases in these inputs will increase milk production. These production increases will become progressively less as input levels increase, until production becomes constant or begins to decrease, beginning stage III of production (10). Increasing feed availability to cows early on will increase milk production, but when the animals reach maximum feed efficiency (i.e., the amount of feed in kilograms required by the animal to produce a liter of milk) (20) their metabolism will be unable to absorb all the nutrients and translate them into greater milk production. These are then expelled in the urine and feces, representing financial losses for producers in the form of costs for excess feed. Increasing the number of cows (ceteris paribus) would reduce the availability of resources such as feed, causing a decrease in total milk production.

Returns to scale for milk production

The milk production homogeneous function exhibited decreasing returns to scale since the sum of the coefficients β₁ and β₂ was 0.888. Therefore a similar percentage increase in all
inputs will cause a percentage increase of smaller magnitude in the product\(^{(1)}\). Similar results have been reported in milk production systems in the state of Sinaloa\(^{(7)}\) in which this effect is attributed to overuse of producer resources and absence of technology use in the system. In this scenario large livestock producers experience increasing returns to scale due to specialization in capital and labor\(^{(8)}\). Presence of this type of return to scale in PU using DP requires evaluation of input use because increasing feed and cow inputs will not raise income from greater production\(^{(21)}\), rather it will cause financial losses due to unnecessary input costs.

**Marginal product value for milk production**

Milk production marginal product results for the feed input indicated that adding 1 kg of feed would increase milk production by 0.75 L, generating additional income of $ 4.03 per unit of added input (*ceteris paribus*). Raising the number of cows in a herd would increase milk production by 892.2 L per year, generating additional income of $ 4,800.20. These results are similar to a study of a DP system in Sinaloa in which marginal product values greater than zero for the cow input caused diminishing marginal returns\(^{(7)}\). This means that increasing herd size to increase milk production is not the best option for improving efficiency in DP systems. Rather, a better approach is to make optimal use of the inputs that have the greatest impact on production. This is supported by the present marginal product values for the feed and cow inputs: both indicate positive economic benefits in producers, but at values less than 1. The law of marginal returns would classify these as diminishing marginal returns, placing them in stage II of a classic production function. Continued increases in these inputs will therefore cause the marginal product to continue decreasing until reaching zero, eventually becoming negative and leading to financial losses. Examples of this dynamic include production units in the eastern portion of the state of Yucatan, Mexico\(^{(5)}\), and tropical dairy production systems in India\(^{(9)}\), both of which have negative marginal returns for the feed input, and, even though marginal product values remain positive, milk production no longer increases.

**Technical optimum milk production levels**

\[ Y_1 = 445.69 X_1^{0.346} X_2^{0.542} \text{ subject to } 4.0 X_1 + 1.36 X_2 = 5.38 \]

Using the Lagrange method:

\[ L = 445.69 X_1^{0.346} X_2^{0.542} - \lambda (4.0X_1 + 1.36 X_2 - 5.38) \]
The partial derivative of $L$ for $X_1$ and $X_2$, under a first order condition:

\[
4.0 \lambda = 154.257X_1^{-0.653}X_2^{0.542} \\
1.36 \lambda = 241.64X_1^{0.346}X_2^{-0.457}
\]

Equalizing the partial derivatives for $X_1$ and $X_2$, and substituting $X_2$ in the constraining equation generates the optimum amount for this input.

\[
\frac{154.257X_1^{-0.653}X_2^{0.542}}{241.64X_1^{0.346}X_2^{-0.457}} = \frac{4.0}{1.36}
\]

\[
X_2 = \frac{4.0X_1}{0.229} = 17.41X_1
\]

\[
X_1 = \frac{5.38}{27.677} = 0.19
\]

\[
X_2 = 17.41 \times 0.19 = 3.38
\]

Substituting the $X_1$ and $X_2$ in the Cobb-Douglas function generates the optimum amount of milk produced.

\[
Y_1 = 445.69 \times 0.19^{0.346}(3.38^{0.542}) = 488.97 \text{L}
\]

Livestock producers in the study area attained an optimal milk production of 488.97 L per day, which is equivalent to producing 12.53 L per day per cow, since on average they had 39 cows in production. In semi-intensive systems, cows must produce 35.38 L per day to achieve optimum milk production using a combination of concentrate feed and fodder inputs\(^{(22)}\). The main difference between the DP and semi-intensive systems is that the latter use a larger amount of concentrate.

**Cobb-Douglas production function for meat**

\[
\ln Y_2 = 0.85366 - 0.20523\ln X_1 + 1.11829\ln X_2 \text{ (Equation 4)}
\]

Transformation via antilogarithms results in:

\[
Y_2 = e^{0.85366}X_1^{-0.20523}X_2^{1.11829} \text{ (Equation 5)}
\]

\[
Y_2 = 2.348X_1^{-0.20523}X_2^{1.11829} \text{ (Equation 6)}
\]
Elasticities of production for meat

The meat production function (Equation 6) shows that a 1% change in the number of cows would increase calf production by 1.11 %, while the same change in the feed input would decrease it by 0.20 % (ceteris paribus). In DP systems, calf feeding is based on controlled lactation in the form of one quarter of the milk in the udder at milking and the quantity and quality of forage consumed when grazing (23). Nutritional supplementation of calves with good quality diets does improve weaning weight in DP systems, but this does not increase the prices paid to the producer for calves. Producers therefore search for alternatives to reduce the weaning period through supplementation with alternative forages (e.g. forage trees and bushes) that improve pre- and post-weaning calf development (24). The present results indicate that the amount of concentrate feed included in calf diets should be reduced because it does not improve overall production performance. Use of concentrate feed generally increases productive variables in livestock production systems (25), although any improvements will depend on feed quantity and quality, since lack of data on the appropriate amount of feed can generate unnecessary costs and cause financial losses for producers.

Marginal returns and input production stage for meat production

The elasticity coefficient for the meat production function is negative for the feed input \((X_1)\), but greater than one for the cow input \((X_2)\) (Equation 4). Values greater than one indicate increasing marginal returns and that the input is in stage I of a classical production function (10). Therefore, increasing the cows input would increase milk production at this stage, making it unadvisable for the producer to lower this input and consequently slow or stop production. Similar behavior has been reported in grazing systems in the State of Mexico and Yucatan (8,9). In other words, increasing herd size within the resources available to the studied PU would raise yield as represented by production variables. In contrast, the feed input exhibited an elasticity of less than zero, representing negative marginal returns and placing it in stage III (10). That is, increasing the amount of feed does not benefit calf production, and indeed could decrease it. A similar effect has been reported for beef production Yucatan, where increases in the amount of concentrate feed did not improve production (9). Rather, a more effective way of increasing calf weaning weight was to properly manage existing pastures by using high quality forages that meet animal nutritional requirements. If the feed input is in stage III of production, the livestock producer is not economically viable because it is spending money on an input that does not increase income from increased production.
Returns to scale for meat production

The meat production functions exhibited diminishing returns to scale because $\sum b_1$ is less than one. That is, increasing all inputs in the same proportion would not increase total production, an effect similar to that reported in small- and medium-sized producers in the State of Mexico\(^8\). Large producers, in contrast, attain increasing returns to scale through genetic improvement (capital) and management efficiency strategies (labor)\(^8\). An alternative for improving returns to scale for small and medium producers using DP would be to increase adoption of technology to improve efficiencies.

Marginal product and marginal product value for meat production

The marginal product of the feed input for meat production was less than zero (Table 4), indicating that total calf production would no longer increase, and that maximum production was attained with a smaller amount of feed. In the studied DP systems this input was used excessively, generating losses of 0.38 cents for each additional unit of feed. In contrast, increasing the number of cows in production by one unit generates $2,460.93 incomes, and because it is in stage I of the production function, the marginal product would continue to increase, as would its value. The same trend has been reported in PU in the state of Yucatan in which increases of one animal unit raised meat production to 980.7 kg, which is attributed to their production being less than maximum due to the limited use of breeding programs and genetic improvement\(^9\). Considering this, the producers studied here need not stagnate calf production by maintaining the cows input unchanged since by increasing this input they could enter stage II of production.

<table>
<thead>
<tr>
<th>Function</th>
<th>Unit Price $</th>
<th>Feed</th>
<th>Cows</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Milk</td>
<td>Calves</td>
<td>PMg</td>
</tr>
<tr>
<td>Milk</td>
<td>5.38</td>
<td>--------</td>
<td>0.75</td>
</tr>
<tr>
<td>Meat</td>
<td>--------</td>
<td>6020</td>
<td>-6.38exp(^{-05})</td>
</tr>
</tbody>
</table>

PMg= marginal product; VPMg= value of marginal product.

Table 4: Marginal product and marginal product value of inputs used in a double-purpose milk and meat production system
Technical optimum meat production level

\[ Y_2 = 2.34 \cdot X_1^{-0.205} \cdot X_2^{1.118} \quad \text{subject to} \quad 4.0 \cdot X_1 + 500 \cdot X_2 = 6,020 \]

By the Lagrange method:

\[ L = X_1^{-0.205} \cdot X_2^{1.118} - \lambda (4.0 \cdot X_1 + 500 \cdot X_2 - 6,020) \]

Partial derivative of L for \( X_1 \) and \( X_2 \), under first order condition:

\[ 4.0 \lambda = 0.481 \cdot X_1^{-1.205} \cdot X_2^{1.188} \]
\[ 500 \lambda = 2.625 \cdot X_1^{-0.205} \cdot X_2^{-0.188} \]

Equalizing partial derivatives for \( X_1 \) and \( X_2 \):

\[ \frac{0.481 \cdot X_1^{-1.205} \cdot X_2^{1.188}}{2.625 \cdot X_1^{-0.205} \cdot X_2^{-0.188}} = \frac{4.0}{500} \]

\[ X_2 = \frac{4.0 \cdot X_1}{91.61} = 0.043 \cdot X_1 \]

Substituting \( X_2 \) in the constraining equation produces the optimum level for this input, and substituting the \( X_1 \) and \( X_2 \) values in the Cobb-Douglas function produces the optimum amount of milk produced.

\[ X_1 = \frac{6,020}{25.5} = 236.07 \]
\[ X_2 = 0.043 \cdot (233.07) = 10.151 \]
\[ Y_2 = 2.348 \cdot (236.07)^{-0.205} \cdot (10.151)^{1.118} = 10.22 \text{ calves} \]

The technical optimum meat production level in the studied PU was 10.22 calves annually. Combining the \( X_1=236.07 \) and \( X_2=10.15 \) inputs maximizes the calf production isoquant.

Conclusions and implications

Feed and cows are the inputs that best explained milk and meat production in dual-purpose livestock producers in the studied areas. Producers need to place more emphasis on their use of these inputs since irrational use can decrease production variables. The elasticities of production indicated that, \textit{ceteris paribus}, increasing these inputs in milk and meat production raises total production, except for the feed input in meat production. In milk
production both inputs exhibited diminishing returns to scale, and were in stage II of production, the stage during which emphasis is needed on production. In meat production, the feed input had a negative marginal product value, placing it in stage III of a production function, and generating financial losses; operating in this stage will cause losses. Although in milk production both inputs had positive marginal product values they should not be increased since they are operating within the law of diminishing marginal returns. The studied livestock producers have generally diminishing returns to scale. One alternative for improving these returns is to increase the use of technology in different areas (feed, pasture management, forage conservation, reproduction, technical training) with the purpose of specializing capital and labor in these systems.

**Literature cited:**


